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G. G. Chetverikov, V. V. Shlyakhov
Kharkiv National University of Radio Electronics, Kharkov

COMPARATOR IDENTIFICATION OF OPERATORS ON HILBERT SPACES FOR BOUNDED INPUT SIGNALS

Г. Г. Четвериков, В. В. Шляхов
Харьковский национальный университет радиоэлектроники (ХНУРЭ), г. Харьков
Харьков, 61166, пр. Науки, 14

ИДЕНТИФИКАЦИЯ ОПЕРАТОРОВ В ГИЛЬБЕРТОВЫХ ПРОСТРАНСТВАХ КОМПАРАТОРНЫМ СПОСОБОМ ПРИ ОГРАНИЧЕНИЯХ ВХОДНЫХ СИГНАЛОВ

Г. Г. Четверіков, В. В. Шляхов
Харківський національний університет радіоелектроніки (ХНУРЕ), м. Харків

ІДЕНТИФІКАЦІЯ ОПЕРАТОРІВ В ГІЛЬБЕРТОВОМУ ПРОСТОРІ КОМПАРАТОРНИМ СПОСОБОМ ПРИ ОБМЕЖЕННЯХ ВХІДНИХ СИГНАЛІВ

It is known that any n-ary relation induces a factorization procedure on its carrier. This is due to the fact that some elements of the carrier are "not recognized". Thus equivalence classes arise. Special cases of n-ary relations are binary and ternary ones. The paper is devoted to the study of conditions, under which binary and ternary relations on equivalence classes induce an algebraic structure in the form of a linear space.
Keywords: predicate, discrete system, equivalence classes, algebra, algebraic structure

Известно, что любое n-арное отношение индуцирует процедуру факторизации на своем носителе. Это происходит в силу того, что некоторые элементы носителя отношением «не распознаются». Таким образом возникают классы эквивалентностей. Частным случаем n-арных отношений являются бинарные и тернарные. Работа посвящена изучению условий, при которых бинарные и тернарные отношения на классах эквивалентностей индуцируют алгебраическую структуру в виде линейного пространства.
Ключевые слова: предикат, дискретная система, классы эквивалентностей, алгебра, алгебраическая структура

Відомо, що будь-яке n-арне ставлення індукує процедуру факторизації на своєму носії. Це відбувається в силу того, що деякі елементи носія ставленням "нерозпізнані". Таким чином виникають класи еквівалентності. Окремим випадком n-арних відносин є бінарні і тернарні. Робота присвячена вивченню умов, при яких бінарні і тернарні відносини на класах еквівалентностей індукують алгебраїчну структуру у вигляді лінійного простору.
Ключові слова: предикат, дискретна система, класи еквівалентності, алгебра, алгебраїчна структура

INTRODUCTION

In many practical situations a set of input signals of an object under study represents a certain algebraic structure. This is explained that, as a rule, the set's elements have definite relations to each other; these relations can be treated as algebraic operations [1]. Correct recognition of a proper structure substantially defines the adequacy of mathematical model [2]. Within the frames of the comparator identification this recognition must be carried out in the language of experimentally testable properties of relations or predicates. For any object a group of its properties acts as structurally organized system of interrelated elements. This defines a scale deformation as multilinear one. It should be also noted that in this paper the operator domain is represented by linear space.

Skipping the details of the experimental part, which is beyond the scope of the paper, we provide a theoretical solution of this problem for such algebraic structure as linear space over a certain field. Such a structure is widely applied in practice [3-5].

The fundamental basis of every modeling as of intuitive analoging is represented by identity relation [6]. It is known that there are several types of identity relations according to their formal characteristics (e.g. isomorphism, homomorphism). Model classification, by generating the sides of the original, stipulates a type of identity to be detected in accordance with its informal characteristics. According to its informal characteristics the model-original identity can be substantial, functional and structural one. The latter, notably the concrete algebraic structure in the form of a linear (Hilbert) space, is the quintessence of the paper.

Problem statement. Suppose the input signal processing system realizes four predicates given on the corresponding Cartesian power of a set M (input signals). They are one unary predicate $P(x)$, one binary predicate $E(x,y)$ and two ternary predicates $S(x,y,z), T(x,y,z)$. The symbols x,y,z denote input signals of the system. The system's output signals are the elements 0 and 1, which are the values of indicated predicates.

Data Predicate Model in the Form of a Linear Space

The predicate $P(x)$ forms coefficients' preimage classes that can be taken in place of the coefficients. $E(x,y)$ is an equivalence predicate given on $M \times M$. It forms vectors' preimage classes that can be taken in place of the vectors. The predicate $S(x,y,z)$ is given on P^3, it defines the addition of coefficients. The predicate $T(x,y,z)$ is given on $P \times M \times M$, it defines the multiplication of coefficients by a vector. Consider the set M with given relations $E(x,y), S(x,y,z), P(x), T(x,y,z)$ satisfying the following properties:

1) $E(x,x)=1;$
2) $E(x,y)=1 \Rightarrow E(y,x)=1;$
3) $E(x,y)=1, E(y,z)=1 \Rightarrow E(x,z)=1;$
4) $\forall x,y \exists\ z : S(x,y,z)=1;$
5) $S(x,y,z)=1, S(x,y,z')=1 \Rightarrow E(z,z')=1;$
6) $S(x,y,z)=1, S(x,y',z)=1 \Rightarrow E(y,y')=1;$
7) $S(x,y,z)=1, S(x',y,z)=1 \Rightarrow E(x,x')=1;$
8) $S(x,y,z)=1 \Rightarrow S(y,x,z)=1;$
9) $S(x,y,z)=1, E(z,z')=1 \Rightarrow S(x,y,z')=1;$

10) $S(x,y,z)=1, E(y,y')=1 \Rightarrow S(x,y',z)=1;$

11) $S(x,y,z)=1, E(x,x')=1 \Rightarrow S(x',y,z)=1;$

12) $S(x,y,z)=1, S(z,t,r)=1, S(y,t,p)=1 \Rightarrow$
$\Rightarrow S(x,p,r)=1;$

13) $\exists 0 : S(x,y,x)=1 \Rightarrow E(y,o)=1;$

14) $\forall x \exists\ (-x) : S(x,-x,y)=1 \Rightarrow E(y,0)=1;$

15) $P(0)=1;$

16) $P(x)=1, P(y)=1, S(x,y,z)=1 \Rightarrow P(z)=1;$

17) $P(x)=1, E(x,y)=1 \Rightarrow P(y)=1;$

18) $\forall x, y \exists\ z : P(x)=1 \Rightarrow T(x,y,z)=1;$

19) $P(x)=0, P(y)=0 \Rightarrow T(x,y,z)=0;$

20) $P(x)=1, P(y)=1, T(x,y,z)=1 \Rightarrow P(z)=1;$

21) $T(x,y,z)=1 \Rightarrow T(x,y,z)=1;$

22) $T(x,y,z)=1, T(x,y,z')=1 \Rightarrow E(z,z')=1;$

23) $T(x,y,z)=1, T(x,y',z)=1 \Rightarrow E(y,y')=1;$

24) $T(x,y,z)=1, T(x',y,z)=1 \Rightarrow E(x,x')=1;$

25) $T(x,y,z)=1, T(z,z')=1 \Rightarrow T(x,y,z')=1;$

26) $T(x,y,z)=1, T(y,y')=1 \Rightarrow T(x,y',z)=1;$

27) $T(x,y,z)=1, T(x,x')=1 \Rightarrow T(x',y,z)=1;$

28) $T(x,y,z)=1, P(x)=1, E(z,0)=1 \Rightarrow E(x,0)=1;$

29) $E(z,0)=1, T(x,y,z)=1 \Rightarrow E(z,0)=1;$

30) $T(x,y,z)=1, P(x)=1, P(y)=1, T(z,p,r)=$
$=1, T(y,p,t)=1 \Rightarrow T(x,t,r)=1;$

31) $T(x,y,z)=1, T(x',y,z')=1, S(x,x',t)=1,$
$P(x)=P(x')=1, S(z,z',p)=1 \Rightarrow T(t,y,p)=1;$

32) $P(x)=1, T(x,y,z)=1, T(x,y',z')=1, S(z,z',t)=1,$
$S(y,y',p)=1 \Rightarrow T(x,p,t)=1;$

$\exists 1 : P(1)=1, T(y,x,x)=1 \Rightarrow E(1,y)=1;$

33) $P(x)=1\ \ \exists x^{-1} : T(x,x^{-1},y)=1 \Rightarrow E(1,y)=1;$

34) $\exists\ \{t_i\}_{i-1}^{n} : \forall x\ \ \exists \{y_i(x)\}_{i=1}^{n} :$ a) $P(y_i(x))=1;$

b) $T(y_i(x), t_i, z_i)=1,$
$S(z_1,z_2,r_1)=1, S(z_1,z_3,r_2)=1, \ldots, S(z_{n-2},z_n,r_{n-1})=$
$=1 \Rightarrow E(x,r_{n-1})=1;$

c) $\forall \{h_i,(x)\}_{i=1}^{n}$, satisfying a) and b) $\Rightarrow E(h_i, y_i(x))=1.$

35) $P(x)=1, P(z)=1, S(x,y,z)=1 \Rightarrow P(y)=1.$

In this case the set M is divided by the relation $E(x,y)$ into equivalence classes. Denote the equivalence classes by $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{R}, \mathrm{T}, \ldots,$ and the set of classes by N. Then, as it was shown in the paper [4], the expression of $E(x,y)$ has the form

$$E(x,y)=D(Fx,Fy),$$

where D is an equality predicate on $\mathrm{N} \times \mathrm{N},$ and $F : M \to N$ (moreover $Fx = Fy \Leftrightarrow E(x,y)=1$).

Our task is to show that the given relations induce the structure of an n-ary linear space by equivalence classes.

Proposition 1. If we define (addition) operation on the equivalence classes as $\mathrm{A}+\mathrm{B}=\mathrm{C}$ if and only if $\forall x,y,z$:

$$x\in\mathrm{A},\ y\in\mathrm{B},\ z\in\mathrm{C},\ S(x,y,z)=1,$$

then the definition is unambiguous and N forms an abelian group with respect to this operation.

Proof. First show that the definition is unambiguous. Take two arbitrary equivalence classes $\mathrm{A},\mathrm{B}\in\mathrm{N}$ and two representatives of each class $x\in\mathrm{A}, y\in\mathrm{B}$. Then taking into account Property 4), we obtain $z\in\mathrm{C}$ such that $S(x,y,z)=1$. This means that $\mathrm{A}+\mathrm{B}=\mathrm{C}$. Therefore, the operation is uniquely determined for any pair $\mathrm{A},\mathrm{B}\in\mathrm{N}$. Indeed, let $\mathrm{C}'\neq\mathrm{C}$, and

$$\mathrm{A}+\mathrm{B}=\mathrm{C},\ \mathrm{A}+\mathrm{B}=\mathrm{C}'.$$

Then for an arbitrary $z'\in\mathrm{C}'$ we have $S(x,y,z')=1$. Considering $S(x,y,z)=1$, and Property 5), we obtain $E(z,z')=1$ or $z'\in\mathrm{C}$. Thus we have $\mathrm{C}\cap\mathrm{C}'\neq\varnothing$; and since distinct classes have empty intersection, it follows that $\mathrm{C}\in\mathrm{C}'$. There is a contradiction. Now we shall show that the class Z is independent of alternatives $x\in\mathrm{A}$ and $y\in\mathrm{B}$. Suppose $x,x'\in\mathrm{A}$ and $y,y'\in\mathrm{B}$; then taking into account Property 11) and since $S(x,y,z)=1$ and $E(x',x)=1$ we obtain $S(x',y,z)=1$. Further, using Property 10) and equality $E(y',y)=1$, we get $S(x',y',z)=1$, This means that the operation of addition is independent of choice of elements from the classes A and B. Thus, the operation is well defined.

Show that N forms an abelian group with respect to this operation.

Suppose $\mathrm{A}+\mathrm{B}=\mathrm{C}$. Then $S(x,y,z)=1$ is true for any $x\in\mathrm{A},y\in\mathrm{B},z\in\mathrm{C}$. In this case, taking into account Property 8), we obtain $S(x,y,z)=1$ or $\mathrm{A}+\mathrm{B}=\mathrm{C}$. Thus

$$\mathrm{A}+\mathrm{B}=\mathrm{B}+\mathrm{A}$$

Consequently, the operation is commutative.

It is associative as well. Let $(\mathrm{A}+\mathrm{B})+\mathrm{C}=\mathrm{R}, \mathrm{A}+\mathrm{B}=\mathrm{T}, \mathrm{B}+\mathrm{C}=\mathrm{G}$. Then for representatives of the classes the equalities $S(x,y,t)=1, S(y,z,g)=1, S(t,z,r)=1$ are true. Using Property 12) we get $S(x,g,r)=1$. This means that $\mathrm{A}+\mathrm{G}=\mathrm{R}$ or $\mathrm{A}+(\mathrm{B}+\mathrm{C})=\mathrm{R}$, i.e.

$$(\mathrm{A}+\mathrm{B})+\mathrm{C}=\mathrm{A}+(\mathrm{B}+\mathrm{C})$$

Hence the operation is associative.

Consider Property 13). It claims that there exists $O\in M$ such that $S(x,O,x)=1$ is true for any x. Therefore, $\mathrm{A}+O=\mathrm{A}$ (O is an equivalence class that contains a unique O). If there is $O'\neq O$, then we obtain $S(x,y,z)=1$ for $y\in O'$ and, taking into account Property 13), we get $E(y,O)=1$, that is $O'=O$. Thus for N there exists a unique element O. This element acts as zero with respect to the operation.

Finally note that there is an inverse element. Let us choose an arbitrary class A and its representative $x\in\mathrm{A}$. Then, it follows from Property 14) that there exists $-x$ such that for $S(x,-x,y)=1$ we have $E(y,0)=1$. Suppose $-x\in-\mathrm{A}$, then $\mathrm{A}+(-\mathrm{A})=\mathrm{B}$, where $y\in\mathrm{B}$, but considering $E(y,0)=1$ we obtain $y\in 0$ or $\mathrm{B}=0$. Hence

$$\mathrm{A}+(-\mathrm{A})=0,$$

where $-\mathrm{A}$ is unique. If the equality is defined for any other class C, then $S(x,z,0)=1, S(x,-x,y)=1$ and $E(y,0)=1$. Therefore, considering Property 9) we obtain $S(x,-x,0)=1$, and from Property 6) we have $-E(-x,z)=1$, i.e. $-x\in\mathrm{C}$ or $-\mathrm{A}=\mathrm{C}$.

The proposition is proved.

Proposition 2. Relation $P(x)$ given on M determines the subset M', which is a union of equivalence classes, where sets of classes in M' form a subgroup of the group of all classes with respect to the addition operation.

Proof. To prove the first part of the lemma statement we shall show that for any equivalence class S the following condition hold: $\mathrm{A}\cap\mathrm{M}'$ is equal to either empty set or A. Indeed, suppose $x\in\mathrm{A}$, then if $P(x)=1$ and $E(x,y)=1$, from Property 17) it follows that $P(y)=1$, i.e. $\mathrm{A}\subset\mathrm{M}'$. If $P(x)=0$, then for any $y\in\mathrm{A}$ we have $P(y)=0$. In the converse case, using Property 17), for $P(y)=1, E(x,y)=1$ we get $P(x)=1$. There is a contradiction. This means that if $P(x)=0$, then $\mathrm{A}\cap\mathrm{M}'=\varnothing$. Thus $\mathrm{M}`=\{x:P(x)=1\}$ is a union of equivalence classes. Denote by N' a set of these classes. Let us prove that $\mathrm{N}'\subset\mathrm{N}$ is a subgroup with respect to the addition operation of classes. Suppose $\mathrm{A},\mathrm{B}\in\mathrm{N}'$, and $\mathrm{A}+\mathrm{B}=\mathrm{C}$. $P(x)=1, P(y)=1$, and $S(x,y,z)=1$. Property 16) states that $P(z)=1$, consequently $z\in\mathrm{N}'$. This means that the addition operation is defined by N'. By Property 15), where $P(0)=1$, we have $0\in\mathrm{N}'$. Finally we shall show that the inverse element belongs to N'. For this reason consider $\mathrm{A}\in\mathrm{N}'$ and $-\mathrm{A}$. Suppose $-\mathrm{A}$ is not contained in $\mathrm{N}`$. Then $P(-x)=0, P(x)=1, P(0)=1$ and $S(x,-x,0)=1$. But the latter set of equalities contradicts Property 36). The proposition is proved.

Proposition 3. If we define (multiplication) operation on the equivalence classes of N' as $\mathrm{AB}=\mathrm{C}$ if and only if $T(x,y,z)=1$ for $\forall x\in\mathrm{A}, y\in\mathrm{B}, z\in\mathrm{C}$, then the product is well defined and the equivalence classes $\mathrm{N}`$ form a field with respect to these operations of addition and multiplication.

Proof. We prove that the operation is well defined in the following way. From Property 18) and Property 20) it follows that for any $\mathrm{A},\mathrm{B}\in\mathrm{N}'$ and their arbitrary elements $x\in\mathrm{A}$ and $y\in\mathrm{B}$ there exists z such that $T(x,y,z)=1$ and $P(z)=1$. Consequently, by definition of multiplication operation, there is a class $\mathrm{C}\subset\mathrm{N}'$ such that $\mathrm{AB}=\mathrm{C}$ and $y\in\mathrm{B}$. Indeed, suppose the equality $T(x,y,z')=1$ holds for some z'. But then by Property 22) and the equality $T(x,y,z)=1$ we have $E(z,z')=1$. This implies that $z'\in\mathrm{C}$. On the other hand, if we choose $x\neq x'\in\mathrm{A}$ and $y\neq y'\in\mathrm{B}$, then by the equalities $E(x,x')=1, E(y,y')=1$ and $T(x,y,z)=1$ as well as by Property 26) and Property 27) we obtain $T(x',y',z)=1$. Therefore, the class C is independent of the initial choice of elements from classes A and B. Consequently, the product of multiplication operation is well defined. Now let us show that a set of classes N' forms a field with respect to the operations of addition and multiplication. From Proposition 7.2 it follows that N' is an abelian group w.r.t. addition. Prove that N' is also an abelian group w.r.t. multiplication. Consider two arbitrary classes $\mathrm{A},\mathrm{B}\in\mathrm{N}'$ and let $\mathrm{AB}\in\mathrm{C}$. The latter equality means that $T(x,y,z)=1$ for $x\in\mathrm{A}, y\in\mathrm{B}, z\in\mathrm{C}$. However, by Property 21) in this case we get $T(x,y,z)=1$ that is $\mathrm{BA}=\mathrm{C}$. Hence, the operation is commutative. From Property 30) it follows that the operation is associative. Indeed, consider $(\mathrm{AB})\mathrm{C}$ and let $\mathrm{AB}=\mathrm{R}, \mathrm{RC}=\mathrm{T}$, $\mathrm{BC}=\mathrm{P}$. Then for representatives of these classes the following equalities hold $T(x,y,z)=1, T(r,z,t)=1, T(y,z,p)=1$. But from Property 30) we have $T(x,p,t)=1$, i.e. $(\mathrm{AB})\mathrm{C}=\mathrm{A}(\mathrm{BC})$.

Let us consider Property 33). It states that for $\forall x\in\mathrm{A}\subset\mathrm{N}'$ there exists $x^{-1}\in\mathrm{A}^{-1}\subset\mathrm{N}'$ such that for any $z\in\mathrm{C}$ we have $T(x,x^{-1},y)=1$ and $T(y,z,z)=1$. If $z\in\mathrm{M}'$, than the latter two equalities determine that $(AA^{-1})C = C,$ besides by Property 20) $AA^{-1} = B \in N'$. We shall show that A^{-1} is independent of the class A. Indeed, suppose $x_1 \neq x$ and $x, x_1 \in A,$ then we have

$$E(x_1, x) = 1, T(x, x^{-1}, y) = 1, T(x_1, x_1^{-1}, y_1) = 1, T(y, z, z) = \\ = 1, T(y_1, z, z) = 1.$$

From these equalities and taking into account Properties 25), 27), 24) we obtain

$$E(y, y_1) = 1, T(x_1, x^{-1}, y) = 1, T(x_1, x^{-1}, y) = \\ = 1, T(x_1, x^{-1}, y_1) = 1$$

And $E(x_1^{-1}, x^{-1}) = 1,$ that is $x_1^{-1} \in A^{-1}$. Let $A_1 = A_2$, and $A_1A_1^{-1} = B_1$, and $A_2A_2^{-1} = B_2$. Then $T(x_1, x_1^{-1}, y_1) = 1,$ $T(x_2, x_2^{-1}, y_2) = 1,$ and by Property 32) we get $\forall z : T(y_1, z, z) = T(y_2, z, z) = 1.$ Now, using Property 24), we have $E(y_1, y_2) = 1,$ consequently $B_1 = B_2$. Therefore, for any $A \subset N' : AA^{-1} = B$ is independent of A. Furthermore, since for $\forall z \in N'$ there is $(AA^{-1})C = C,$ then AA^{-1} acts as identity w.r.t. multiplication. Consequently, we denote $AA^{-1} = E.$

Finally we see, that sets of classes N' form groups w.r.t. the operations of multiplication and addition. In addition, the interrelation of these operations defines N' to be a field. Let us show this. Actually we prove all axioms of a field except of distributivity. This axiom follows from Property 31). For representatives of arbitrary classes $A, A', B, C, C', T, P \subset N'$ by equalities $AB = C, A'B = C, A + A' = T, C = C = P$ we get $T(x, y, y) = T(x', y, z') = S(x, x', t) = S(z, z', p) = 1$, and from Property 31) we have $T(t, y, p) = 1$ that is $TB = P,$ consequently $AB + A'B = (A + A')B.$ This implies that distributivity holds and evidently completes the proof of the proposition.

Let us summarize the results of proved propositions. The defined relations

1) divide an initial set into equivalence classes;

2) these equivalence classes form a set $N,$ inducing the operation of addition, with respect to this operation the set N is a group.

3) In the set N there exists a subset $N' \subset N,$ on which the initial relations induce the operation of multiplication. With respect to the operations of multiplication and addition the set N' is a field.

Now we shall define and prove the theorem as the object of this part of the paper.

Theorem 1. The set of equivalence classes N is a finite-dimensional linear space over the field N' with vector addition defined in the Proposition 1, and multiplication (defined in Proposition 3) of a vector by an element of the field.

Proof. First note that the multiplication operation induced by the relation T for elements of the field $N',$ similarly to Proposition 3, can be well-defined for elements $N,$ that is the multiplication of vectors by elements of the field N`. The proof of this fact is similar to the proof of Proposition 3. Let us prove the Theorem.

It was already shown that the set of elements (further we shall call them vectors, but denote by capital letters since they are equivalence classes) N form a group. Also, we have the field N and the multiplication of elements of the field by a vector. Show that this operation has the following properties:

1) if $A \in N', B, C \in N,$ then $A(B + C) = AB + AC;$

2) if $A, B \in N', C \in N,$ then $(AB)C = A(BC);$

3) if $\mathrm{A},\mathrm{B}\in \mathrm{N}',\mathrm{C}\in \mathrm{N},$ then $(\mathrm{A}+\mathrm{B})\mathrm{C}=\mathrm{AC}+\mathrm{BC};$

4) for $O,E\in \mathrm{N}'$ there exists $O\mathrm{A}=O$, and there is $E\mathrm{A}=\mathrm{A}$ for an arbitrary $\mathrm{A}\in \mathrm{N}.$

The first one follows from Property of relations 32), as this is shown in Proposition 3. The third and the second properties are actually proved in Proposition 3, when considering associativity and distributivity of the operations of addition and multiplication. Let us consider property 4), namely that $E\mathrm{A}=\mathrm{A}$ follows from Property of relations 33). To define the second equality we can use Property 29): To prove the second equality we can use Property 29): if we consider $O\mathrm{A}=\mathrm{C},$ then for elements we have $y'\in O$ and $S(z,-z,y)=1,$ where if we consider $O\mathrm{A}=\mathrm{C},$ then for the elements $y'\in O$ and $S(z,-z,y)=1$ we get $E(y',y)=1,$ and then, $T(y',x,t)=1,$ that is $O\mathrm{A}=\mathrm{T}$ and $E(y,t)=1$ or $\mathrm{T}=O.$ It means that $O\mathrm{A}=O.$

Thus, we show that the set N is a linear space over the field $N'.$ Let us prove that it is a finite-dimensional one.

By Property 35) we can get elements $t_1\in T_1,...,t_n\in T_n$ such that for any $x\in \mathrm{A}$ there exist unique $y_1(x)\in \mathrm{B}_1(\mathrm{A}),...,y_n(\mathrm{A})\in \mathrm{B}_n(\mathrm{A}),$ where the following conditions hold:

$A)\ P(y_i)=1$ that is $\mathrm{B}_i(\mathrm{A})\in \mathrm{N}$ are elements of the field;

$B)\ T(y_i(x),t_i,z_i)=1$ that is $\mathrm{B}_i(\mathrm{A})T_i=Z_i;$

$S(z_1,z_2,r_1)=1$ that is $\mathrm{C}_1+\mathrm{C}_2=\mathrm{R}_1$, etc.;

$S(r_{n-2},z_n,r_{n-1})=1,$ i.e. $\mathrm{R}_{n-2}+\mathrm{C}_n=\mathrm{R}_{n-1}$ or

$\mathrm{C}_1+\mathrm{C}_2+...+\mathrm{C}_n=\mathrm{R}_{n-1}.$

Then from Property 35) it follows that $E(x,r_{n-1})=1,$

consequently, $R_{n-1}=\mathrm{A}.$ Finally we obtain basis expansion $T_1,...,T_n$

$\mathrm{A}=\mathrm{B}_1(\mathrm{A})T_1+...+\mathrm{B}_n(\mathrm{A})T_n.$

The uniqueness of classes $\mathrm{B}_i(\mathrm{A})$ (as opposed to the elements $y_i(x)$ considered in Property 35)) follows from point b) of Property 35). The theorem is proved.

Conclusion

The axiomatics of a multialgebraic system existence in the form of a linear space is found. We actually obtained necessary and sufficient conditions for binary and ternary relations to induce the structure of a linear space with the elements represented by equivalence classes over an arbitrary field.

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RESUME

G. G. Chetverikov, V. V. Shlyakhov

Comparator Identification of Operators on Hilbert Spaces for Bounded Input Signals

Background: such algebraic structure as linear space over a certain field is widely applied in practice. The fundamental basis of every modeling is an identity relation. Model classification by generating the sides of the original stipulates a type of identity to be detected in accordance with its informal characteristics. According to its informal characteristics the model-original identity can be substantial, functional and structural one. The latter, notably the concrete algebraic structure in the form of a linear (Hilbert) space, is the quintessence of the paper.

Materials and methods: Suppose the input signal processing system realizes four predicates given on the corresponding Cartesian power of a set M (input signals). They are one unary predicate $P(x)$, one binary predicate $E(x,y)$ and two ternary predicates $S(x,y,z), T(x,y,z)$. The symbols x,y,z denote input signals of the system. The system's output signals are the elements 0 and 1, which are the values of indicated predicates.

Results: The given relations induce the structure of an n-ary linear space by equivalence classes. The set of equivalence classes N is a finite-dimensional linear space over the field N' with vector addition defined in the Proposition 1, and multiplication (defined in Proposition 3) of a vector by an element of the field.

Conclusion: The axiomatics of a multialgebraic system existence in the form of a linear space is found. Necessary and sufficient conditions for binary and ternary relations to induce the structure of a linear space with the elements represented by equivalence classes over an arbitrary field are obtained.

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